OPERATION RESEARCH

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Syllabus:

Unit - I Linear Programming Problems: Origin and development of operations research,
Linear Programming Problem –formulation of Linear Programming problem, Graphical solution. Theory of simplex method. Use of artificial variables and their solution. [13 Hours]

Unit - II Transportation Problem: Mathematical formulation of transportation problem, Initial basic Feasible solution, North West corner rule, Matrix minima method, Vogel's approximation method, MODI method to find optimal solution. [13 Hours]

Unit - III Assignment Problem: Mathematical formulation of an Assignment problem, Assignment algorithm, Hungarian Method to solve Assignment Problem. [13 Hours]

Unit - IV Network Analysis: Basic components of Network, Rules for drawing Network diagram Time calculation in Networks. Critical Path Method and PROJECT Evaluation and Review Techniques. Algorithm and flow chart for CPM and PERT. [13 Hours]

Unit - V ` Theory of Games: Two –person Zero –sum Games, the maximin and Minimax principle, Saddle point and value of the Game. Game without saddle points, mixed strategies, solution for 2X2 games, Graphical method Dominance property. [13 Hours]

UNIT 1: LINEAR PROGRAMMING PROBLEM

CHAPTER 1: ORIGIN AND DEVELOPMENT OF OR

DEFINITION: Operations research (OR) is an analytical method of problem-solving and decision-making that is useful in the management of organizations. In operations research, problems are broken down into basic components and then solved in defined steps by mathematical analysis.**(2 MARKS)**

Application of operation research (2 MARKS)

1. Purchasing and procurement
2. Production management
3. Research and development
4. Personnel management
5. Marketing
6. finance, budgeting and investment.

Main phases of OR(6 MARKS)

- Definition and formulation of the problem
- Construction of the model
- Solution of the model
- Validation of the model
- Establish control over the solution
- Implementation of the solution

Advantages of OR(2 MARKS)

- Optimum use of factors
- Improved quality of decision
- Preparation of future managers
- Modification of mathematical solution
- Alternative solution.

CHAPTER 2:LINEAR PROGRAMMING PROBLEM

Definition: LPP deals with optimization (maximization/minimization)of a linear function of variables called the **objective function** subjected to a set of linear equation and /or inequalities called the **constraints** or **restriction.(2 M)**

Formulation of linear programming:

- **Objective function(2 M)**
- **Constraints**
- **Non negativity constraints**

Standard form of LPP(4 M)

Maximize $z=c_1x_1+c_2x_2+.....+c_nx_n$

Subject to constraints $a_{11}x_1+a_{12}x_2+.....+a_{1n}x_n=b_1$

$a_{21}x_1+a_{22}x_2+......+a_{2n}x_n=b_2$

.

$a_{m1}x_1+a_{m2}x_2+....+a_{mn}x_n=b_n$

$x_1\geq 0,\ x_2\geq 0,\ ....x_n\geq 0 \qquad b_1\geq 0, b_2\geq 0\ ....b_n\geq 0$

- **Explain the types of solution of a LPP**(4 M)
  ANS:

Solution of LPP: any set $x=\{x_1,x_2,......x_{n+m}\}$ of variables is called a solution of LPP.if it satisfies only the set of given constraints equations.

Basic feasible solution of LPP: A basic feasible solution is a basic solution which also satisfies all basic variables are non-negative.

Optimal solution of LPP : Any feasible solution which optimizes (Min or Max) the objective function of the LPP is called its optimum solution.

Degenerate basic feasible solution: A basic feasible solution is said to degenerate if one or more basic variable are zero.

Non-Degenerate basic feasible solution: A basic feasible solution is said to be non degenerate if all the basic variables are greater than zero.

Slack variable: if a constraint is of <= type , we add a non negative variable called slack variables to the LHS of the constraint.(2 m)

Surplus variable: if the constraint is of >=type, we subtract a non-negative variable called the surplus variable from the LHS of the constraints.

Graphical method:

A simple linear programming problem with two decision variables can be easily solved by the graphical method The steps involved in the graphical method are as follows.

Step 1: Consider each inequality constraint as an equation.

Step 2: Plot each equation on the graph as each will geometrically represent a straight line.

Step 3 :Mark the region. If the inequality constraint corresponding to that line is the region below the line lying in the first quadrant (due to non-negativity of variables) is shaded. For the inequality constraint P sign, the region above the line in the first quadrant is shaded. The points lying in common region will satisfy all the constraints simultaneously. The common region, thus obtained, is called the feasible region.

Step *4*:Assign an arbitrary value, say zero, for the objective function.

Step 5 Draw a straight line to represent the objective function with the arbitrary value (i.e., a straight line through the origin).

Step 6 Stretch the objective function line till the extreme points of the feasible region. In the maximization case, this line will stop farthest from the origin, passing through at least one corner of the feasible region. In the minimization case, this line will stop nearest to the origin and passes through at least one corner of the feasible region.

Step 7 Find the coordinates of the extreme points selected in step 6 and find the maximum or minimum value of Z.

Note: As the optimal values occur at the corner points of the feasible region, it is enough to calculate the value of the objective function of the corner points of the feasible region and select the one which gives the optimal solution, i.e., in the case of maximization problem the optimal point corresponds to the corner point at which the objective function has a maximum value and in the case of minimization, the corner point which gives the objective function the minimum value is the optimal solution

Example 2.9: Solve the following LPP by graphical method.

Minimize $Z = 20X_1 + 10X_2$

Subject to $X_1 + 2X_2 \leq 40$

$3X_1 + X_2 \geq 30$

$4X_1 + 3X_2 \geq 60$

$X_1, X_2 \geq 0$

Solution: Replace all the inequalities of the constraints by equation

$X_1 + 2X_2 = 40$ If $X_1 = 0 \Rightarrow X_2 = 20$

If $X_2 = 0 \Rightarrow X_1 = 40$

$\therefore X_1 + 2X_2 = 40$ passes through (0, 20) (40, 0)

$3X_1 + X_2 = 30$ If $X_1 = 0 \Rightarrow X_2 = 30$

If $X_2 = 0 \Rightarrow X_1 = 10$

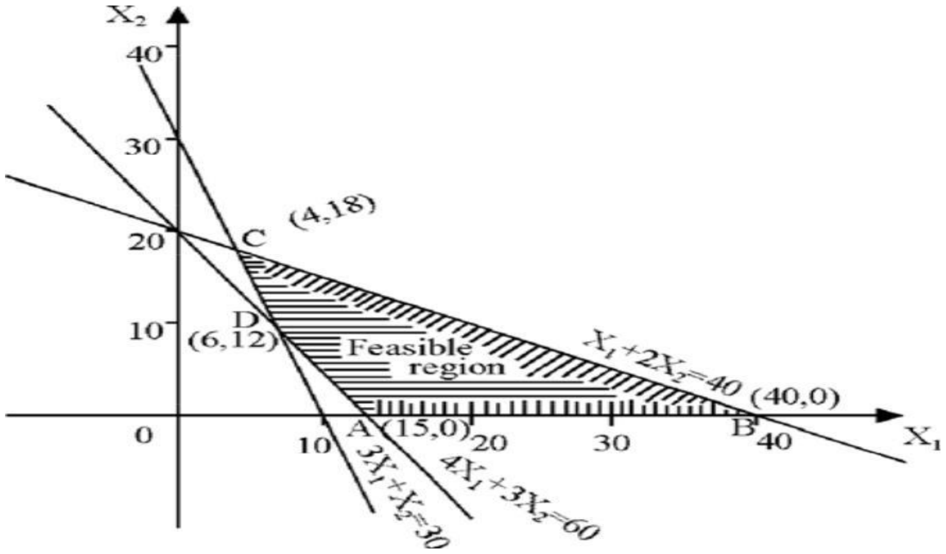
$3X_1 + X_2 = 30$ passes through (0, 30) (10, 0)

$4X_1 + 3X_2 = 60$ If $X_1 = 0 \Rightarrow X_2 = 20$

If $X_2 = 0 \Rightarrow X_1 = 15$

$4X_1 + 3X_2 = 60$ passes through (0, 20) (15, 0)

Plot each equation on the graph.



The feasible region is *OABCD.*

B and C are points of intersection of lines.

$X_1 + X_2 = 4$, $10X_1 + 7X_2 = 35$ and

$3X_1 + 8X_2 = 24$, $X_1 + X_2 = 4$

On solving we get,

B = (1.6, 2.3)

C = (1.6, 2.4)

Corner points	*Value of Z* $=5X_1 + 7X_2$
O (0, 0)	0
A (3.5, 0)	17.5
B (1.6, 2.3)	25.1
C (1.6, 2.4)	24.8 (Maximum value)

D (0, 3)	21

∴ The maximum value of Z occurs at *C* (1.6, 2.4) and the optimal solution is X_1=1.6, X_2 = 2.4.