UNIT I: <u>Number Systems</u>

The Natural Numbers

The **natural** (or **counting**) **numbers** are $1,2,3,4,5,$1,2,3,4,5, etc.

There are infinitely many natural numbers.
The set of natural numbers, $\{1,2,3,4,5,...\}${1,2,3,4,5,...}, is written $\mathbb{N}$.

The **whole numbers** are the natural numbers together with 0.

The sum of any two natural numbers is also a natural number
(for example, $4+2000=2004$4+2000=2004)

the product of any two natural numbers is a natural number
($4\times2000=8000$4×2000=8000).

This is not true for subtraction and division, though.

The Integers

The **integers** are the set of real numbers consisting of the natural numbers, their additive inverses and zero.

$\{...,-5,-4,-3,-2,-1,0,1,2,3,4,5,...\}${...,−5,−4,−3,−2,−1,0,1,2,3,4,5,...}
The set of integers is written $\mathbb{Z}$.

The sum, product, and difference of any two integers is also an integer.
But this is not true for division.

The Rational Numbers

The **rational numbers** are those numbers which can be expressed as a ratio between two integers.

All the integers are included in the rational numbers.

The Irrational Numbers

An **irrational number** is a number that cannot be written as a ratio (or fraction). In decimal form, it never ends or repeats.

The Real Numbers

The real numbers is the set of numbers containing all of the rational numbers and all of the irrational numbers. The real numbers are “all the numbers” on the number line. There are infinitely many real numbers just as there are infinitely many numbers in each of the other sets of numbers.

HIGHEST COMMON FACTOR:

The Highest Common Factor (HCF) of two or more integers is the largest positive integer that divides the numbers without a remainder.

For example, the HCF of 8 and 12 is 4.

Prime Factorisations

Highest Common Factor can be calculated by first determining the prime factors of the two numbers and then write down the common factors. Then HCF is product of least power of common factors.

example: HCF (18, 42),

first prime factors of

18 = 2 * 3 * 3

42 = 7 * 2 * 3

the “common” of the two expressions is 2 * 3;

So HCF (18, 42) = 6.

– By Division Method

In this method first divide a higher number by smaller number.

- Put the higher number in place of dividend and smaller number in place of divisor.
- Divide and get the remainder then use this remainder as divisor and earlier divisor as dividend.
- Do this until you get a zero remainder. The last non zero remainder is the HCF.
- If there are more than two numbers then we continue this process as we divide the third lowest number by the last divisor obtained in the above steps.
-

First find H.C.F. of 72 and 126

72|126|1
72
54| 72|1
54
18| 54| 3
54
0

H.C.F. of 72 and 126 = 18

LEAST COMMON MULTIPLE:

The Least Common Multiple of two or more integers is the integer which is divisible by both of the given numbers.

Prime Factorizations

The prime factorization theorem says that every positive integer greater than 1 can be written in only one way as a product of prime numbers.

Example: To find the value of LCM (9, 48, and 21).

First, find the factor of each number and express it as a product of prime number powers.

Like 9 = 3^2,
48 = 2^4 * 3
21 = 3 * 7

Then, write all the factors with their highest power like 3^2, 2^4, and 7. And multiply them to get their LCM.
Hence, LCM (9, 21, and 48) is $3^2 * 2^4 * 7 = 1008$.

Division Method
Here, divide all the integers by a common number until no two numbers are further divisible. Then multiply the common divisor and the remaining number to get the LCM.

2 | 72, 240, 196
2 | 36, 120, 98
2 | 18, 60, 49
3 | 9, 30, 49
3 | 3, 10, 49
5 I1,10,49
7 I 1,2,49
1,2,7

L.C.M. of the given numbers
= product of divisors and the remaining numbers
= 2×2×2×3×3×5x7x7x2x7
= 72×10×49 = 35280.
Relation between L.C.M. and H.C.F. of two natural numbers
The product of L.C.M. and H.C.F. of two natural numbers = the product of the numbers.
For Example:

LCM (8, 28) = 56 & HCF (8, 28) = 4

Now, 8 * 28 = 224 and also, 56 * 4 = 224

HCF & LCM of fractions:
Formulae for finding the HCF & LCM of a fractional number.

HCF of fraction = HCF of numerator / LCM of denominator
LCM of Fraction = LCM of Numerator / HCF of Denominator